Int. J Heat Mass Transfer. Vol. 10, pp. 1159 1170. Pergamon Press Ltd. 1967. Printed in Great Britain

HEAT TRANSFER AND BINARY DIFFUSION IN VARIABLE PROPERTY FREE CONVECTION FLOWS

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(*Received* 10 *May* 1966 *and in revised form* 3 *December* 1966)

Abstract—Variable property free convection stagnation flow and free convection on a vertical flat plate are analyzed for the case of simultaneous momentum, heat and binary mass transfer with thermodynamic coupling. The helium–air and hydrogen–air systems are considered in detail. A simplified general treatment of diffusion thermo effects is developed and applied to obtain approximate but accurate expressions for evaluating heat-transfer rates and adiabatic wall temperatures. This method shows why the driving force based on $(T_w - T_{aw})$ can be used to correlate heat-transfer results for situations where diffusion thermo is important. It is significant that the method also provides a simple error estimate for the correlation obtained by using the adiabatic wall temperature.

It is found that free convection stagnation flow and free convection on a vertical plate are very similar in nature when compared on the proper basis. This seems interesting since, in the vertical plate case one employs the boundary-layer approximations whereas the stagnation flow represents an exact solution of the Navier–Stokes equations.

A correlation of the σ function, which enables one to calculate heat-transfer rates for both free convection configurations studied here, both hydrogen and helium mixtures with air, and over the range of T_w/T_e in which diffusion thermo is important, is given and appears to be accurate enough for almost all engineering purposes.

NOMENCLATURE

C,	heat capacity at constant [e.g. cal/g C];
D,	coefficient of diffusion;
DT,	diffusion thermo;
F,	stream function, equation (9);
h,	heat-transfer-coefficient;
j,	mass flux by diffusion, equation (1);
k,	thermal conductivity;
M,	molecular weight;
Nu,	Nusselt number, equation (13);
Pr,	Prandtl number, $\mu C/k$;
R,	universal gas constant;
r,	radius of cylinder;
r'_q,	defined by equation (3d);
Sc,	Schmidt number, $\mu/D\rho$;
T,	temperature, absolute;
TD,	thermal diffusion;
u,	x-component of velocity;
U_e,	velocity at edge of boundary layer;
v,	y-component of velocity;
x,	distance along surface;
y,	distance normal to surface.

Greek symbols

α,	thermal diffusion factor;
γ,	variable defined by equation (14);
Γ,	variable defined by equation (8);
Γ_2,	variable defined by equation (21);
Δ,	variable defined by equation (6);
η,	variable defined by equation (9);
θ_1,	$(T - T_e)/(T_w - T_e)$;
θ_2,	variable defined by equation (17);
θ_{DT},	variable defined by equation (17);
Λ,	denotes ratio of property to its value in free stream, $\Lambda_\rho = \rho/\rho_e$, $\Lambda_\mu = \mu/\mu_e$, $\Lambda_k = (k/k_e)$;
$\Lambda_{C_{AB}}$,	denotes $(C_A - C_B)/C_e$;
μ,	viscosity;
ν,	kinematic viscosity;

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ρ, density;
ϕ, $(\omega_A - \omega_{A_e})/(\omega_{A_w} - \omega_{A_e})$;
ψ, stream function, equation (9);
Ψ, $= \frac{\Lambda_{k_w}}{\Lambda_{\mu_w}} \theta_2'(0) \left(\frac{T_w}{T_e} + 1\right)^{\frac{1}{4}}$
ω, mass fraction;
Ω, variable defined by equation (7);
σ, function defined by equation (24).

Subscripts
av, arithmetic average, $(T_e + T_w)/2$;
aw, refers to condition of adiabatic wall;
A, refers to component A, injected gas;
B, refers to component B, ambient gas;
x, refers to x-direction;
y, refers to y-direction;
w, refers to conditions at surface;
e, refers to condition at edge of boundary layer;
0, no interfacial mass transfer.

INTRODUCTION

In a previous paper [2], the authors considered free convection on a vertical flat plate, wherein fluid properties were assumed to depend on concentration variations only, and thermodynamic coupling effects were ignored. However, recent work [4, 5, 6, 7, 9] has shown that diffusion thermo effects can be important in free convection stagnation flow and that heat-transfer effects in such systems can be correlated, in terms of solutions which neglect coupling effects, by employing the adiabatic wall temperature in the driving force for heat transfer.

The purposes of the present study are to:

(1) Extend our previous work in investigating free convection on a vertical flat plate and account for property dependence on both temperature and concentration as accurately as possible.
(2) Compare free convection on a vertical flat plate with free convection near the stagnation point of a horizontal cylinder. These configurations are interesting from an experimental viewpoint since they can be studied without using a wind tunnel.
(3) Explain why the adiabatic wall temperature enables one to correlate the effects of diffusion thermo and also to provide an error estimate for such a correlation in free convection systems.

The study of binary systems is complicated enormously by the fact that fluid properties can vary radically with both temperature and concentration and this in turn markedly affects heat, mass and momentum transfer. Thus, one must evaluate the transport properties of non-isothermal mixtures and in the present study the method which employs the kinetic theory of gases with the Lennard–Jones model for intermolecular forces was used as outlined by Hirschfelder, Curtiss and Bird [3]. Specific details of the approach are available elsewhere [10].

The variable property equations of change are well known but in most instances one invokes the boundary-layer assumptions to solve them. However, as pointed out by Sparrow *et al.* [5] free convection in the neighborhood of the stagnation point of a horizontal cylinder is one of the rare cases which admits an exact solution to the Navier–Stokes equations without making the boundary-layer simplification. In contrast, free convection of a vertical plate cannot be solved in a straightforward fashion unless one does employ boundary-layer assumptions. Thus, it seems that it should be interesting to compare the results obtained for these two configurations.

ANALYSIS

In the problem considered here, j_A, the mass flux by diffusion, and q, the heat flux are given by

$$j_A = -\rho D\left(\frac{\partial \omega_A}{\partial y} + \frac{\alpha \omega_A(1-\omega_A)}{T}\frac{\partial T}{\partial y}\right) \quad (1)$$

$$q = -k\frac{\partial T}{\partial y} + \alpha R T \frac{M}{M_A M_B} j_A. \quad (2)$$

For $\omega_{A_e} = 0$, the equations of change with variable fluid properties can be written in dimensionless form as

$$\left(\frac{F'}{\Lambda_\rho \Lambda_\mu}\right)'' + F\left(\frac{F'}{\Lambda_\rho \Lambda_\mu}\right)' - \beta \Lambda_\rho \Lambda_\mu \left(\frac{F'}{\Lambda_\rho \Lambda_\mu}\right)^2 + E(\eta) = 0 \qquad (3)$$

$$\left(\frac{\phi'}{Sc}\right)' + F\phi' + \Delta' = 0 \qquad (4)$$

$$\left(\frac{\Lambda_k}{\Lambda_\mu}\theta_1'\right)' + \Omega \frac{\Lambda_k}{\Lambda_\mu}\theta_1' + \frac{1}{(T_w/T_e) - 1}\Gamma' = 0 \qquad (5)$$

where

$$\Delta = \frac{\alpha\phi(1 - \omega_{A_w}\phi)}{Sc\{1 + [(T_w/T_e) - 1]\theta_1\}}\left(\frac{T_w}{T_e} - 1\right)\theta_1' \qquad (6)$$

$$\Omega = Pr_e\left\{\Lambda_C F + \omega_{A_w}\Lambda_{C_{AB}}\left(\frac{\phi'}{Sc} + \Delta\right)\right\}\frac{\Lambda_\mu}{\Lambda_k} \qquad (7)$$

$$\Gamma = \frac{\omega_{A_w}\alpha R\{1 + [(T_w/T_e) - 1]\theta_1\}}{M_A} \times \frac{M}{M_B}\frac{\mu_e}{k_e}\left(\frac{\phi'}{Sc} + \Delta\right) \qquad (8)$$

if one uses the following transformations for both free convection on a vertical plate (FCVP), and near the stagnation point of a horizontal cylinder (FCS):

FCVP

$$\beta = \tfrac{2}{3},\ E(\eta) = \Lambda_\mu(1 - \Lambda_\rho),$$

$$\eta = \left(\frac{3}{4}\frac{g}{\nu_e^2 x}\right)^{\frac{1}{4}} \int_0^y \Lambda_\mu^{-1}\,\mathrm{d}y,$$

$$\psi = \frac{4}{3}\left(\frac{3}{4}\frac{g\nu_e^2}{x}\right)^{\frac{1}{4}} xF(\eta),$$

FCS $\qquad (9)$

$$\beta = 1,\ E(\eta) = \Lambda_\mu(1 - \Lambda_\rho)$$

$$\eta = \left(\frac{g}{\nu_e^2 r}\right)^{\frac{1}{4}} \int_0^y \Lambda_\mu^{-1}\,\mathrm{d}y$$

$$\psi = \left(\frac{g_0 \nu_e^2}{r}\right)^{\frac{1}{4}} xF(\eta).$$

In all cases the boundary conditions are

$$\eta = 0: \quad F = F(0),\ F'(0) = 0),\ \phi = \theta_1 = 1 \qquad (10)$$

$$\eta \to \infty: \quad F'(\infty) = 0,\ \phi \to 0,\ \theta_1 \to 0, \qquad (11)$$

and the wall concentration is determined from the coupled condition

$$F(0) = \frac{\omega_{A_w}}{1 - \omega_{A_w}}\left[\frac{\phi'(0)}{Sc} + \Delta_w\right]. \qquad (12)$$

RESULTS

The magnitude of the effects of diffusion thermo on heat transfer is indicated in Fig. 1 wherein the ratio of the heat flux at the wall with blowing q_w, to that with no blowing, q_{w_0}, is plotted vs. the blowing rate for a helium–air system. It is seen that the heat transfer from a surface at a temperature greater than that of the free stream may be substantially reduced, and even reversed, whereas heat transfer to a

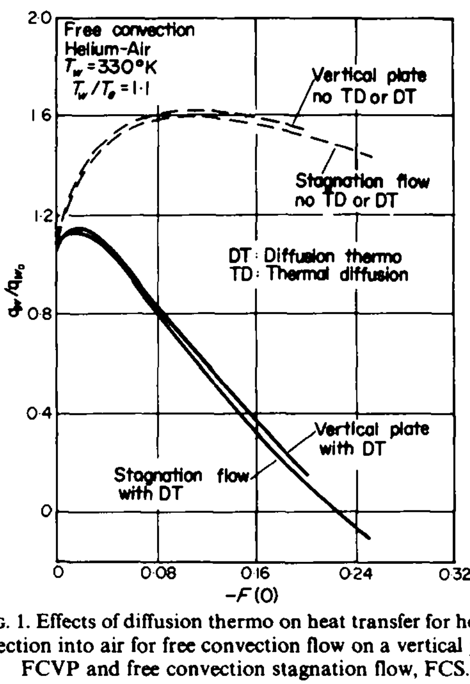

FIG. 1. Effects of diffusion thermo on heat transfer for helium injection into air for free convection flow on a vertical plate, FCVP and free convection stagnation flow, FCS.

(FCVP): $\quad -F(0) = v_w \Lambda_{\rho_w}[(4/3)(x/\nu_e^2 g)]^{\frac{1}{4}}$,

(FCS): $\quad -F(0) = v_w \Lambda_{\rho_w}[r/g\nu_e^2]^{\frac{1}{4}}$.

surface at a temperature below that of the free stream is increased. Furthermore, it is interesting to note that the behavior of the heat flux ratio as a function of $F(0)$ is very similar for the two free convection systems considered even though the field force is constant for FCVP and linear in distance along the surface for FCS.

Comparison of heat-transfer results based on exact solutions with others in the literature

The Nusselt number is defined as

$$Nu = \frac{hx}{k_e} = \frac{q_w x}{k_e \Delta T} \tag{13}$$

and, letting the driving force, ΔT, be given by $T_w - T_{aw}$, we can then write

$$Nu_x \gamma(x) = \left[\frac{(T_w/T_e) - 1}{(T_w/T_e) - (T_{aw}/T_e)}\right] \frac{\Lambda_{k_w}}{\Lambda_{\mu_w}} \theta_1'(0) + \frac{\Gamma(0)}{(T_w/T_e) - (T_{aw}/T_e)} \tag{14}$$

where $\gamma(x) = [\frac{4}{3}(\nu_e^2/gx^3)]^{\frac{1}{4}}$ for free convection vertical plate flow and $\gamma = (\nu_e^2/gr^3)^{\frac{1}{4}}$ for free convection stagnation flow.

Heat-transfer data suitable for comparison with those of the present study have been reported only for FCS. No comparable work, either mathematical or experimental, on FCVP is known to the authors. The quantity defined by equation (14) is plotted vs. the blowing parameter in Fig. 2 where both helium and hydrogen injection are considered. Included in the figures are the theoretical solutions of this study for FCS, as well as those of Sparrow *et al.* [5], and the experimental results of Tewfik *et al.* [9] and Sparrow *et al.* [7]. In the case of helium injection the present analytical results are in slightly better agreement with the experiments of Tewfik than are the calculations of Sparrow. However, the analytical results of Sparrow agree slightly better with his own experimental data. In the case of hydrogen injection the present solutions agree very well with those of Sparrow

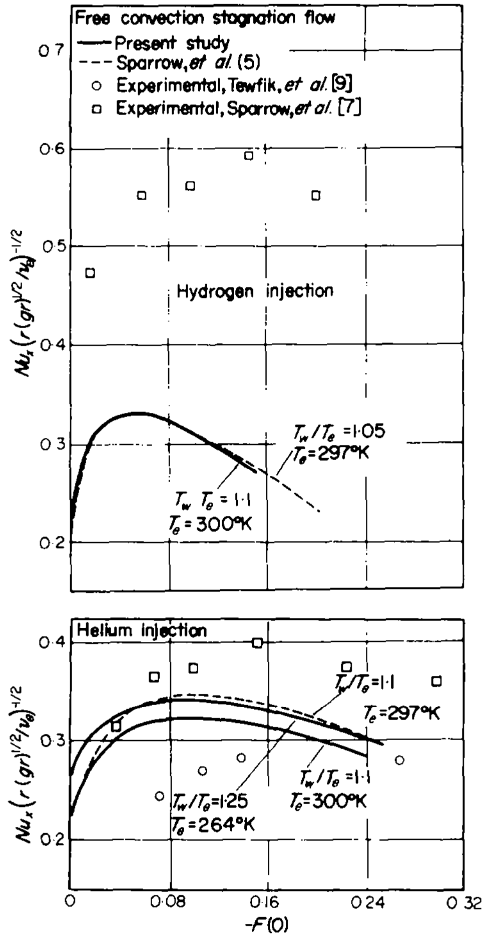

FIG. 2. Nusselt number defined in terms of the adiabatic wall temperature vs. $F(0)$, for hydrogen and helium injection into air in free convection stagnation flow.

et al., but neither agrees well with the experimental data available. There is at present no adequate explanation for the discrepancy between theory and experiment for the hydrogen data.

The small deviations between the present mathematical results and those of Sparrow *et al.* for the case of helium injection are probably due to the different experimental physical property data used to evaluate the force constants as discussed in reference [10]. This statement is supported by the fact that agreement is much better for the case of hydrogen injection. Furthermore, in view of the much larger differences between experimental studies the small differences between the theoretical studies seem rather unimportant.

Examination of the effects of diffusion thermo on heat transfer

It has been pointed out in several investigations [4–6] that under ordinary conditions the effect of thermal diffusion on the mass flux, j_A, has a negligible effect on the heat transferred, and that the effects of diffusion thermo on the heat-transfer coefficient are fairly small if the coefficient is defined in terms of the difference between the wall temperature and the adiabatic wall temperature. Since the heat-transfer process in binary systems is so complicated and involves a large number of parameters these discoveries are important. Therefore we shall attempt to explain in rather simple fashion why the use of the adiabatic wall temperature enables one to correlate heat-transfer results in terms of solutions which neglect thermodynamic coupling completely, and also how large an error one can expect to incur when using this concept.

If the effect of thermal diffusion on j_A is ignored, equations (7) and (8) simplify to

$$\Omega = Pr_e \left\{ \Lambda_C F + \omega_{A_w} \Lambda_{C_{AB}} \frac{\phi}{Sc} \right\} \frac{\Lambda_\mu}{\Lambda_k} \qquad (15)$$

$$\Gamma = \frac{R\mu_e}{k_e} \omega_{A_w} \alpha \left[1 + \left(\frac{T_w}{T_e} - 1 \right) \theta_1 \right] \times \frac{M}{M_A M_B} \frac{\phi'}{Sc}. \qquad (16)$$

To a first approximation Λ_k/Λ_μ and Sc are independent of temperature and the thermal diffusion factor and heat capacity are rather slowly varying functions of temperature. The only strongly temperature dependent quantity in the diffusion thermo term is

$$1 + \left(\frac{T_w}{T_e} - 1 \right) \theta_1 = \frac{T}{T_e}.$$

However, diffusion thermo is important only when T_w/T_e differs from unity by less than a factor of two and in this range $|T/T_e - 1|$ is on the order of $\frac{1}{2}$, or less. Also, in this range, we can diminish the effect of T/T_e on the diffusion thermo term and thereby make this term "more nearly" an inhomogeneity by dividing the diffusion thermo term by the arithmetic average temperature ratio across the boundary layer. We can accomplish this and also eliminate the quantity $R\mu_e/k_e$ from the diffusion thermo term by defining

$$\theta_1 = \theta_2 + \frac{1}{2} \left(\frac{T_w}{T_e} + 1 \right) \left(\frac{1}{(T_w/T_e) - 1} \right) \times \frac{R\mu_e}{k_e} \theta_{DT} \qquad (17)$$

where θ_2, which is the exact variable property solution obtained by neglecting thermodynamic coupling completely, satisfies

$$\left(\frac{\Lambda_k}{\Lambda_\mu} \theta_2' \right)' + \Omega \frac{\Lambda_k}{\Lambda_\mu} \theta_2' = 0 \qquad (18)$$

$$\theta_2(0) = 1, \qquad \theta_2(\infty) = 0 \qquad (19)$$

and θ_{DT} satisfies

$$\left(\frac{\Lambda_k}{\Lambda_\mu} \theta_{DT}' \right)' + \Omega \frac{\Lambda_k}{\Lambda_\mu} \theta_{DT}' + \Gamma_2 = 0 \qquad (20)$$

$$\Gamma_2 = \alpha \omega_{A_w} \frac{M}{M_A M_B} \frac{2\left[1 + \left(\frac{T_w}{T_e} - 1 \right) \theta_1 \right]}{T_w/T_e + 1} \frac{\phi'}{Sc} \qquad (21)$$

$$\theta_{DT}(0) = 0, \qquad \theta_{DT}(\infty) = 0. \qquad (22)$$

Clearly, the term $R\mu_e/k_e$ has been eliminated from Γ but note that

$$\frac{2\left[1 + \left(\frac{T_w}{T_e} - 1 \right) \theta_1 \right]}{\frac{T_w}{T_e} + 1} = \frac{T/T_e}{T_{av}/T_e}$$

remains and it depends explicitly on T_w/T_e. As mentioned previously in situations where diffusion thermo is important the temperature does not vary very significantly from T_e and thus temperature variations in Γ_2, over the range of T_w/T_e in which diffusion thermo is important, do not markedly alter the solutions for θ_{DT}. Furthermore, with very good accuracy, one can take $\theta_1 \simeq \theta_2$ in Γ_2 and then equation (20) can be solved independently. However, in free convection systems the momentum equation, via the body force term, is highly sensitive to T_w/T_e and this causes θ_{DT} to vary substantially with T_w/T_e as shown on Figs. 3 and 4.

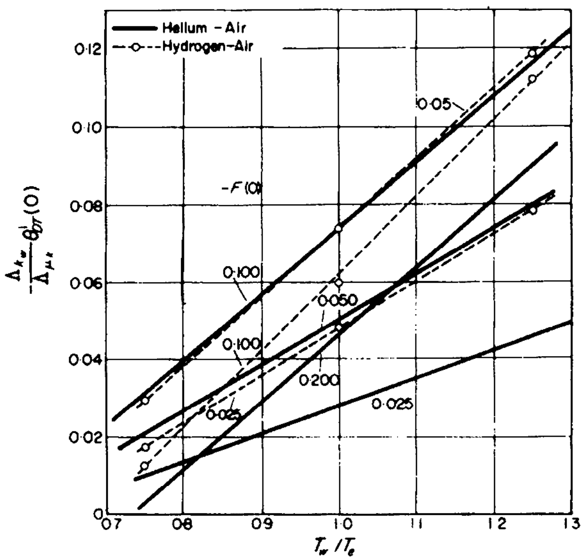

FIG. 3. Variation of the diffusion thermo function with the wall to free stream temperature ratio, T_w/T_e, with the mass flux, $-F(0)$ as parameter for free convection on a vertical plate.

A simple method of correlating free convection heat-transfer results can be developed rather easily for fairly large blowing rates where buoyancy effects are due primarily to concentration gradients. At very low blowing rates free convection is created primarily by temperature gradients and the type of correlation to be presented is not nearly as effective. In the limit of no mass transfer, the effects of temperature dependent fluid properties can be accounted for quite well by the method given by Sparrow and Gregg [8].

The quantity of primary concern here is the heat flux at the wall, which is given by

$$q_w = -k_w \frac{\partial T}{\partial y}\bigg|_w - \alpha_w R T_w \frac{M_w}{M_A M_B}\rho_w D_w \frac{\partial \omega_A}{\partial y}\bigg|_w$$

or

$$\begin{aligned} q_w x y(x) &= -T_e k_e \left[\left(\frac{T_w}{T_e} - 1\right)\frac{\Lambda_{k_w}}{\Lambda_{\mu_w}}\theta_1'(0)\right. \\ &\quad \left. + \frac{1}{2}\left(\frac{T_w}{T_e} + 1\right)\frac{R\mu_e}{k_e}\Gamma_2(0)\right] \\ &= -T_e k_e \left[\left(\frac{T_w}{T_e} - 1\right)\frac{\Lambda_{k_w}}{\Lambda_{\mu_w}}\theta_2'(0)\right. \\ &\quad \left. + \frac{1}{2}\left(\frac{T_w}{T_e} + 1\right)\frac{R\mu_e}{k_e}\left\{\frac{\Lambda_{k_w}}{\Lambda_{\mu_w}}\theta_{DT}'(0) + \Gamma_2(0)\right\}\right] \\ &= -T_e k_e \left(\frac{T_w}{T_e} - 1\right)\frac{\Lambda_{k_w}}{\Lambda_{\mu_w}}\theta_2'(0) \\ &\quad \times \left[1 - \frac{\left(\frac{T_w}{T_e} + 1\right)}{\left(\frac{T_w}{T_e} - 1\right)}\frac{1}{2}\frac{R\mu_e}{k_e}\sigma\right] \end{aligned} \tag{23}$$

where

$$\sigma = -\frac{\frac{\Lambda_{k_w}}{\Lambda_{\mu_w}}\theta_{DT}'(0) + \Gamma_2(0)}{\frac{\Lambda_{k_w}}{\Lambda_{\mu_w}}\theta_2'(0)}. \tag{24}$$

If one can now assume that the temperature dependence of the physical properties, as they appear in equations (3), (4), (18) and (20), can be adequately represented in terms of θ_2 only, (which is a good approximation), one can solve equations (3), (4), (18) simultaneously and then use the results to solve equation (20). Fortunately, despite the variation of θ_{DT} with T_w/T_e, it will be found, in the range where diffusion thermo is important, that this effect can be reported in terms of σ functions, defined by equation (24) and which are reasonably insensitive to T_w/T_e. Consequently the separation used in equation (17) is quite useful.

It would be desirable to obtain simple correlations of $(\Lambda_{k_w}/\Lambda_{\mu_w})\,\theta_2'(0)$ and σ for both of the gases, H_2 and He, and both of the configurations, FCVP and FCS, considered in this

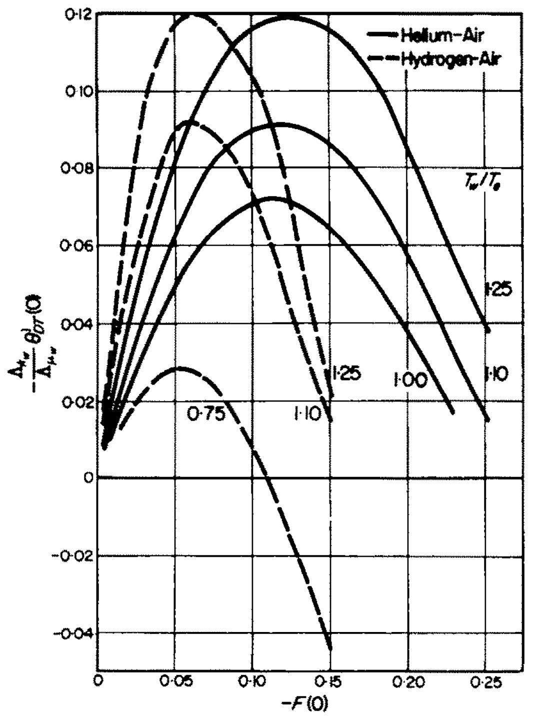

FIG. 4. Variation of the diffusion thermo function with mass flux, $-F(0)$, with the wall to free stream temperature ratio as parameter for free convection stagnation flow on a horizontal cylinder.

study. First, consider the correlation of σ which is shown on Fig. 5. It is seen that the combination of terms used to define σ is a good one since this quantity behaves quite well, over the entire range of T_w/T_e in which diffusion thermo is important, when plotted vs. the molal wall flux $[-F(0)/M_A]$.

The correlation in Fig. 5 applies to both of the free convection configurations (FCS and FCVP) and to both gases, H_2 and He. Cases where buoyancy forces caused by temperature and concentration differences act in the same direction, $T_w/T_e > 1$, and where they act in opposite directions, $T_w/T_e < 1$, are included in this correlation. In this study of various air mixtures, the results of Sparrow *et al.* [6] showed that of all gases considered only H_2 and He exhibited significant diffusion thermo effects on heat transfer. Therefore, Fig. 5 provides the correlation of free convection heat transfer results for all conditions in which diffusion thermo is known to be important.

The solutions for $(\Lambda_{k_w}/\Lambda_{\mu_w})\,\theta_2'(0)$, however, do not correlate nearly as well as the σ results when plotted against $[-F(0)/M_A]$ since the

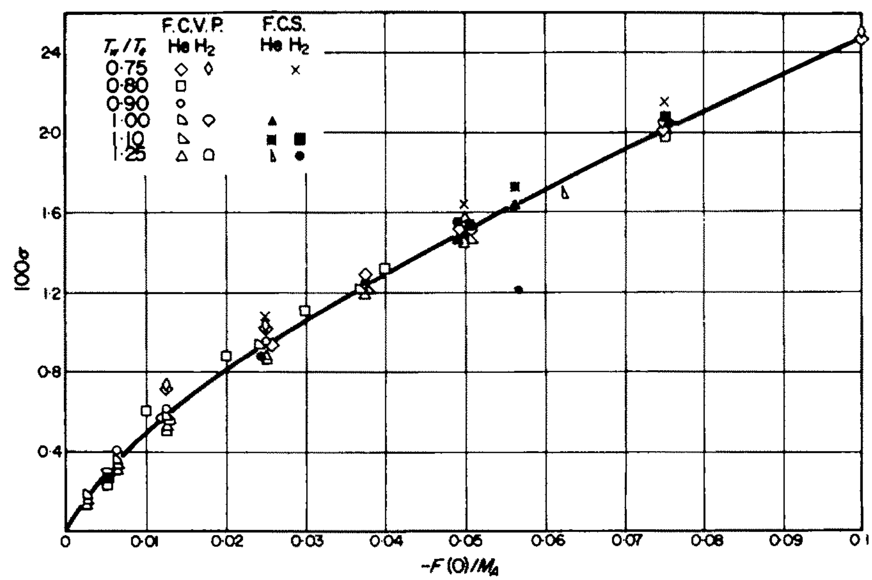

FIG. 5. Correlation of the σ function for various wall to free stream temperature ratios, for hydrogen–air and helium–air mixtures in free convection flow on a vertical plate, FCVP and at the stagnation point of a horizontal cylinder, FCS.

curves are displaced systematically with T_w/T_e. In a number of previous studies variable property results have been correlated successfully, in terms of constant property solutions, by evaluating appropriate physical properties at a suitable reference temperature. Over the range of T_w/T_e studied here, it is found that the quantity $[(T_w/T_e) + 1]^{\frac{3}{4}}$ can be used to scale $(\Lambda_{k_w}/\Lambda_{\mu_w})\,\theta_2'(0)$ effectively. That is, if one plots

$$\frac{\Lambda_{k_w}}{\Lambda_{\mu_w}}\theta_2'(0)\left(\frac{T_w}{T_e}+1\right)^{-\frac{3}{4}}$$

vs. $[-F(0)/M_A]$ as in Figs. 6(a) and 6(b), the results correlate well for $[-F(0)/M_A]$ values of about 0·015 or greater. For very small blowing rates the exponent of $\frac{3}{4}$ is too small but in the limit of no blowing one can use the convenient method discussed by Sparrow and Gregg [8] since the present results suggest that their approach should apply equally well to FCS and FCVP. When T_w/T_e differs markedly from unity the exponent is too large but it applies quite well over most of the region in which diffusion thermo is significant and thus it provides a convenient way to report the results of the present work.

The results have been plotted separately in Figs. 6(a) and 6(b) based on whether buoyancy forces caused by concentration and temperature differences act in the same or opposite directions, $T_w/T_e > 1$ or $T_w/T_e < 1$ respectively. This separation was made primarily for the purpose of reducing the congestion created by having the data points so closely packed. Clearly, at larger blowing rates the bands, representing the data between the two essentially parallel lines on each figure, are very nearly identical. In the large blowing rate region a single line can be used to represent all the data plotted with an error less than 5 per cent.

THE ADIABATIC WALL TEMPERATURE

The adiabatic wall temperature, T_{aw}, is that T_w for which the heat transfer at the wall, q_w, is zero. Thus, from equation (23) one can solve for T_w/T_e such that $q_w = 0$, to get

$$T_{aw} - 1 = -\frac{1}{2}\frac{R\mu_e}{k_e}\left(\frac{T_{aw}}{T_e}+1\right) \times \frac{\dfrac{\Lambda_{k_w}}{\Lambda_{\mu_w}}\theta_{DT}'(0) + \Gamma_2(0)}{\dfrac{\Lambda_{k_w}}{\Lambda_{\mu_w}}\theta_2'(0)} \tag{25}$$

or, rearranging,

$$\frac{T_{aw}}{T_e} = \frac{1+\dfrac{1}{2}R\dfrac{\mu_e}{k_e}}{1-\dfrac{1}{2}R\dfrac{\mu_e}{k_e}\sigma} = 1 + \frac{R\dfrac{\mu_e}{k_e}\sigma}{1-\dfrac{1}{2}R\dfrac{\mu_e}{k_e}\sigma}. \tag{26}$$

It was shown earlier that the σ function is quite insensitive to changes in T_w/T_e, as it is to changes in the temperature level, T_e. Hence, using the correlation in Fig. 5, equation (26) provides a simple method of evaluating the adiabatic wall temperature for arbitrary T_e. It is noteworthy that the results obtained using the σ function in equation (26) are found to be quite accurate, when compared with exact solutions.

It has been shown previously (5) that T_{aw}/T_e decreases substantially with increased T_e, and it appeared that this was caused by an increase in C_e which occurred in the denominator of their diffusion thermo term. Note however, that $Pr_e = (\mu_e C_e/k_e)$ appears in the same term, so that C_e actually drops out, and the decrease in T_{aw}/T_e with increased T_e must be primarily attributed to a decrease in the ratio μ_e/k_e. This is in agreement with the predictions of equation (26) of the present study.

Use of adiabatic wall temperature to correlate the effects of diffusion thermo on heat transfer

It is of some importance to determine the magnitude of the error involved in using the difference between the wall temperatures as a driving force to eliminate the effect of diffusion thermo. If a Nusselt number is defined using

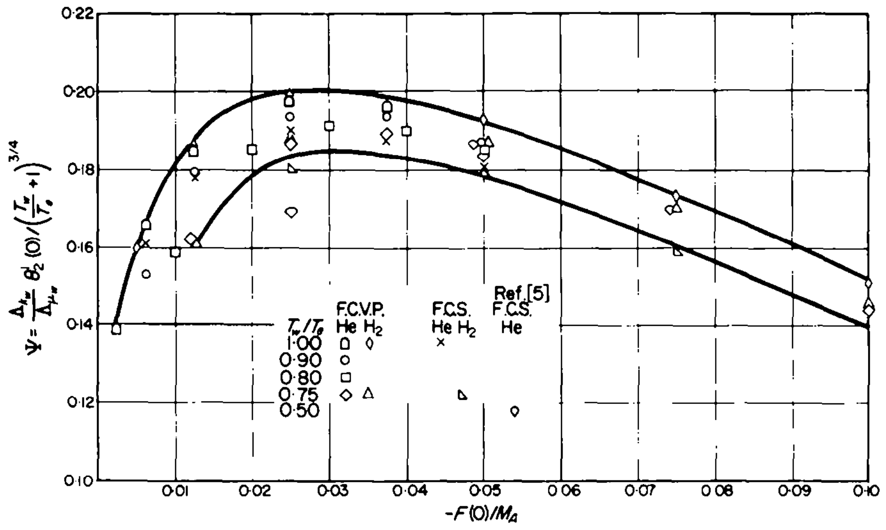

FIG. 6(a). Correlation of the reduced temperature function, Ψ, for various wall to free stream temperature ratios, $T_w/T_e < 1{\cdot}0$ for hydrogen–air and helium–air mixtures in free convection flow on a vertical plate, FCVP, and at the stagnation point of a horizontal cylinder, FCS.

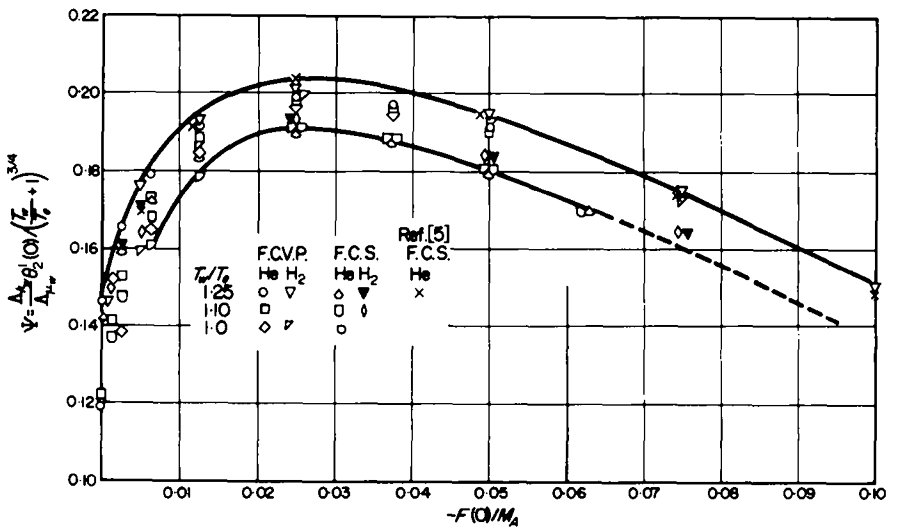

FIG. 6(b). Correlation of the reduced temperature function, Ψ, for various wall to free stream temperature ratios, $T_w/T_e < 1{\cdot}0$ for hydrogen–air helium–air mixtures in free convection flow on a vertical flat plate, FCVP, and at the stagnation point of a horizontal cylinder, FCS.

$T_w - T_{aw}$ as the driving force, as it is in equation (14), then

$$Nu_x\gamma = -\frac{\frac{T_w}{T_e} - 1}{\frac{T_w}{T_e} - \frac{T_{aw}}{T_e}}\left\{\frac{A_{k_w}}{A_{\mu_w}}\theta_2'(0)\right\}$$

$$-\frac{\frac{T_w}{T_e} + 1}{\frac{T_w}{T_e} - \frac{T_{aw}}{T_e}}\frac{1}{2}\frac{R\mu_e}{k_e}\left\{\frac{A_{k_w}}{A_{\mu_w}}\theta_{DT}'(0) + \Gamma_2(0)\right\}$$

$$= -\frac{A_{k_w}}{A_{\mu_w}}\theta_2'(0) - \frac{\frac{A_{k_w}}{A_{\mu_w}}\theta_2'(0)}{\frac{T_w}{T_e} - \frac{T_{aw}}{T_e}}$$

$$\times\left[\left(\frac{T_{aw}}{T_e} - 1\right) - \frac{1}{2}\left(\frac{T_w}{T_e} + 1\right)\frac{R\mu_e}{k_e}\sigma\right]. \qquad (27)$$

Using the relation for T_{aw}/T_e given in equation (26), we then obtain

$$Nu_x\gamma = -\frac{A_{k_w}}{A_{\mu_w}}\theta_2'(0) - \frac{\frac{A_{k_w}}{A_{\mu_w}}\theta_2'(0)}{\frac{T_w}{T_e} - \frac{T_{aw}}{T_e}}\frac{1}{2}\frac{R\mu_e}{k_e}$$

$$\left[\left(\frac{T_{aw}}{T_e} + 1\right)\sigma_{aw} - \left(\frac{T_w}{T_e} + 1\right)\sigma\right] \qquad (28)$$

where σ_{aw} is the σ obtained when the energy equation is solved for the case of an adiabatic wall. It has been shown that σ is primarily a function of the blowing rate. Hence, to a good approximation $\sigma_{aw} \cong \sigma$, and equation (28) becomes

$$Nu_x\gamma = -\frac{A_{k_w}}{A_{\mu_w}}\theta_2'(0) + \frac{1}{2}R\frac{\mu_e}{k_e}\frac{A_{k_w}}{A_{\mu_w}}\theta_2'(0)\,\sigma. \qquad (29)$$

The first term is just that which would be obtained if diffusion thermo were ignored completely in solving the energy equation, and the Nusselt number defined using $T_w - T_e$ as the driving force. The second term represents, to a good approximation, the error involved in assuming that the effects of diffusion thermo are eliminated from the heat-transfer coefficient by defining it in terms of the adiabatic wall temperature, i.e. the error involved in assuming the diffusion thermo term to be an inhomogeneity in the energy equation. It is interesting to note that for a given blowing rate, the percentage error is relatively constant, regardless of either the value of T_w/T_e or the importance of diffusion thermo. For example, equation (29) may be rewritten as

$$Nu_x\gamma = -\frac{A_{k_w}}{A_{\mu_w}}\theta_2'(0)\left[1 - \frac{1}{2}\frac{R\mu_e}{k_e}\sigma\right]. \qquad (30)$$

Since σ, as shown earlier, is essentially independent of the ratio T_w/T_e, and of the temperature level T_e, for a given blowing rate the percentage error is essentially independent of T_w/T_e, and varies with T_e only via the ratio μ_e/k_e. Furthermore, since σ increases with blowing over most of the blowing range, so does the percentage error.

ACKNOWLEDGEMENT

This work was supported by the Office of Naval Research under Contract 5053(01).

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Résumé— L'écoulement au point d'arrêt avec convection naturelle et propriétés variables et la convection naturelle sur une plaque plane verticale sont analysés dans le cas d'un transport simultané de quantité de mouvement, de chaleur et de transport de masse binaire avec couplage thermodynamique. Les systèmes hélium–air et hydrogène–air sont considérés en détail. Un traitement général simplifié des effets thermiques dus à la diffusion est exposé et appliqué pour obtenir des expressions approchées mais précises afin d'évaluer les flux de transport de chaleur et les températures pariétales adiabatiques. Cette méthode montre pourquoi la force motrice basée sur $(T_w - T_{aw})$ peut être employée pour relier ensemble les résultats de transport de chaleur dans les cas où les effets thermiques dus à la diffusion sont importants. Il est important de remarquer que la méthode fournit aussi une estimation simple de l'erreur pour la relation obtenue grâce à la temperature pariétale adiabatique.

On trouve que l'écoulement au point d'arrêt avec convection naturelle et la convection naturelle sur une plaque verticale sont d'une nature très similaire lorsqu'on les compare sur des bases appropriées. Ceci semble intéressant puisque, dans le cas de la plaque verticale on emploie les approximations de la couche limite tandis que l'écoulement au point d'arrêt représente une solution exacte des équations de Navier–Stokes.

On donne une relation de la fonction σ, qui permet de calculer les flux de transport de chaleur pour les deux configurations de convection naturelle étudiées ici, pour les deux mélanges hydrogène et hélium avec l'air, et dans la gamme de T_w/T_e où les effets thermiques dus à la diffusion sont importants. Cette relation semble assez précise pour presque tous les buts techniques.

Zusammenfassung—Die Staupunktströmung bei freier Konvektion mit veränderlichen Stoffwerten und die Strömung entlang einer senkrechten ebenen Platte bei freier Konvektion wurden analysiert für gleichzeitigen Impuls- Wärme- und binären Stofftransport mit thermodynamischer Kopplung. Helium–Luft- und Wasserstoff–Luft-Systeme werden eingehend besprochen. Eine einfache allgemeine Behandlung des Diffusionsthermoeffektes wird entwickelt und dazu angewandt, Näherungsausdrücke zur Berechnung des Wärmeübergangs und der adiabaten Wandtemperatur zu erhalten. Diese Methode zeigt, warum die auf der Differenz $(T_w - T_{aw})$ beruhende treibende Kraft zur Korrelation der Wärmeübergangsergebnisse verwendet werden kann, in Fällen, in denen der Diffusionsthermoeffekt von Bedeutung ist. Die Methode ermöglicht auch eine einfache Fehlerabschätzung für eine Korrelation, die unter Verwendung der adiabaten Wandtemperatur erhalten wurde.

Bei entsprechendem Vergleich zeigt sich, dass die Staupunktströmung bei freier Konvektion und die Strömung entlang einer senkrechten Platte bei freier Konvektion in ihrer Natur sehr ähnlich sind. Dies erscheint interessant, da bei senkrechter Platte die Grenzschichtnäherungen Verwendung finden, während die Staupunktströmung eine exakte Lösung der Navier–Stokesschen Gleichungen zulässt.

Eine Korrelation der σ Funktion zur Berechnung des Wärmeübergangs bei freier Konvektion für beide hier untersuchte Anordnungen bei Wasserstoff- und Heliumgemisch mit Luft wird angegeben für den Bereich von T_w/T_e in dem der Diffusionsthermoeffekt von Bedeutung ist. Sie scheint für nahezu alle Ingenieur-Anwendungen genau genug zu sein.

Аннотация—Проведен анализ свободно-конвективных течений жидкости с переменными свойствами вблизи критической точки и на вертикальной плоской пластине в случае совместного переноса количества движения, тепла и бинарной массы, при наличии термодинамического взаимодействия. Подробно рассмотрены системы гелий–воздух и водород–воздух. Разработан упрощенный метод анализа диффузионного термоэффекта, который дает приближенные, но достаточно хорошие выражения для оценки тепловых потоков и адиабатической температуры стенки. Метод показывает, почему данные по теплообмену, где диффузионный термоэффект играет большую роль, можно обобщить с помощью движущей силы основанной на $(T_w—T_{aw})$. Важно также, что этот метод дает простую оценку погрешности для корреляции, полученной при использовании адиабатической температуры стенки.

Установлено, что свободно-конвективные течения в критической области и на вертикальной пластине аналогичны по своему характеру, если их сравнить соответствуюшим образом ; это интересно т.к. в случае вертикальной стенки используется апрокси-

мация пограничного слоя, тогда как в критической области применяются точное решение уравнений Навье–Стокса.

Приводится обобщенная формула для функции σ, позволяющая рассчитать тепловые потоки при свободной конвекции исследованной геометрии для смесей водорода с воздухом и гелия с воздухом в диапазоне отношений T_w/T_e, в котором диффузионный термоэффект играет значительную роль. Эта формула является достаточно точной для большинства инженерных целей.